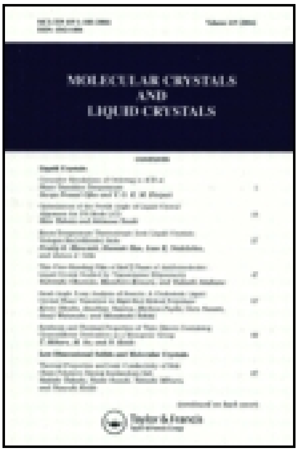

Molecular Crystals and Liquid Crystals
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl20

Quantum Bits with Electric Charges by Josephson Junction

Ryoichi Suzuki [a] , Hiroaki Saito [a] , Tomoya Yamaguchi [a] , Ayumu Sugiyama [a] , Isao Sakamoto [a] , Hidemi Nagao [a] & Kiyoshi Nishikawa [a]

[a] Department of Computational Science, Faculty of Science, Kanazawa University, Kanazawa, 920-1192, Japan

Version of record first published: 18 Oct 2010

Mol. Cryst. Liq. Cryst., Vol. 379, pp. 561-566

1058-725X/02 $12.00 + .00
DOI: 10.1080/10587250290091056

Quantum Bits with Electric Charges by Josephson Junction

RYOICHI SUZUKI, HIROAKI SAITO, TOMOYA YAMAGUCHI, AYUMU SUGIYAMA, ISAO SAKAMOTO, HIDEMI NAGAO and KIYOSHI NISHIKAWA

Department of Computational Science, Faculty of Science, Kanazawa University, Kanazawa 920-1192, Japan

Recently, K.Ch.Chatzisavvas *et al.* have proposed the one- and two-qubit device by means of Josephson junction. In this work, we extend their work to realize the three-qubit device, and construct explicitly the *CCNOT* gate of the three-qubit device.

Keywords Quantum Computer ; Qubit ; Josephson junction

INTRODUCTION

The current computer is based upon the concept of 'bit', the values of which are just 0 and 1. In 1985, Deutsch first showed the concept of the quantum computer, where an arbitrary superposed state of two quantum-levels is considered instead of classical bit[1]. In 1992, Deutsch and Jozsa[2] exhibited a class of problems that can be solved more rapidly on quantum computers than on classical ones, and recently Shor[3] proposed new algorithm and showed that the quantum computer could factor large composite integers very efficiently. For this problem, no efficient classical algorithm is known. Clearly, the experimental realization of quantum computation is a most important issue.

To construct a quantum computer, we have to realize physically a quantum bit or 'qubit', which is the basic component of a quantum computer, and many methods were presented to make theoretically a two-level quantum system. Recently, an artificial qu-

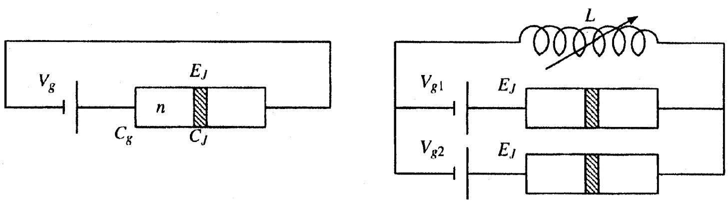

FIGURE 1. One- and two-qubit device

bit was realized by using superconductivity, where Josephson junction constitutes a single-Cooper-pair box, and the action of one-qubit was evidenced by coherent control of macroscopic quantum states[4], where one-qubit and two-qubit device are concluded. In this work, we extend the works done by K.Ch.Chatzisavvas to realize the three-qubit device.

ONE- AND TWO-QUBIT DEVICE

In this section, we briefly summarize the results obtained by K.Ch.Chatzisavvas *et al.*[5], where the device is made of the Josephson junction.

First, we mention the simple Josephson junction one qubit device shown in Figure 1. This device consists of a small superconducting island 'box' with n Cooper pair charges connected by a tunnel junction. and the junction has capacitance C_J and Josephson coupling energy E_J to a superconducting electrode. The voltage V_g is coupled to the system via a gate capacitor C_g .

Now, the superconducting charges box reduces to a two-state quantum system, 'qubit', with Hamiltonian:

$$H = \frac{1}{2}E_c\sigma_z - \frac{1}{2}E_J\sigma_x.$$

This Hamiltonian has two parameters, the bias energy E_c and the tunneling amplitude E_J.

Arbitrary one-qubit gates are 2×2 unitary matrices, and belong to $SU(2)$ group. For example, the projection of command NOT is given by the following correspondence.

$$NOT = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \mapsto i\sigma_x \in SU(2) \quad \left(\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} \right)$$

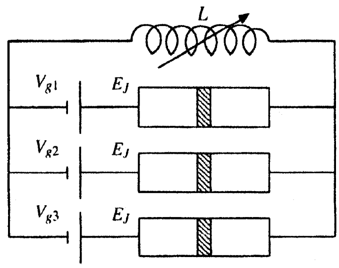

FIGURE 2. Three-qubit device

Next we turn to the two-qubit device. The parameter E_L takes two values. The one is $E_L = 0$ corresponding to a an uncoupled two qubit state and the other one has a fixed value. The Hamiltonian for a general two qubit system is written by:

$$\begin{aligned} H &= H_1 + H_2 + H_{int} \\ &= \frac{1}{2}E_{c_1}\sigma_z^{(1)} - \frac{1}{2}E_{J_1}\sigma_x^{(1)} + \frac{1}{2}E_{c_2}\sigma_z^{(2)} - \frac{1}{2}E_{J_2}\sigma_x^{(2)} - \frac{1}{2}E_L\sigma_y^{(1)}\sigma_y^{(2)} \end{aligned}$$

and the projection of command $CNOT$ is given by

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \mapsto \frac{e^{i\frac{\pi}{4}}}{2}(-\sigma_z^{(1)}\sigma_x(2) + \sigma_z^{(1)} + \sigma_x^{(2)} + I \otimes I) \in SU(4).$$

THREE-QUBIT DEVICE

Here we develop the three-qubit device by extending the previous treatment.

In order to perform three-qubit quantum gate manipulations, we consider the circuit consisting of three Josephson junctions, and a mutual inductor L as shown in Figure 2. The Hamiltonian for a general three-qubit system is written:

$$\begin{aligned} H &= H_1 + H_2 + H_3 + H_{int} \\ &= \frac{1}{2}E_{c_1}\sigma_z^{(1)} - \frac{1}{2}E_{J_1}\sigma_x^{(1)} + \frac{1}{2}E_{c_2}\sigma_z^{(2)} - \frac{1}{2}E_{J_2}\sigma_x^{(2)} + \frac{1}{2}E_{c_3}\sigma_z^{(3)} - \frac{1}{3}E_{J_3}\sigma_x^{(3)} \\ &\quad -\frac{1}{2}E_L\sigma_y^{(1)}\sigma_y^{(2)} - \frac{1}{2}E_L\sigma_y^{(1)}\sigma_y^{(3)} - \frac{1}{2}E_L\sigma_y^{(2)}\sigma_y^{(3)} \end{aligned}$$

$$(\sigma_i^{(1)} = \sigma_i \otimes I \otimes I,\ \ \sigma_i^{(2)} = I \otimes \sigma_i \otimes I,\ \ \sigma_i^{(3)} = I \otimes I \otimes \sigma_i$$
$$\sigma_i^{(1)}\sigma_j^{(3)} = \sigma_i \otimes I \otimes \sigma_j)$$

For example, the projection of command $CCNOT$ is given by

$$CCNOT = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$
$$\mapsto \frac{e^{i\frac{\pi}{8}}}{4}(\sigma_z^{(1)} + \sigma_z^{(2)} + \sigma_x^{(3)} + 3 \cdot I \otimes I \otimes I$$
$$-\sigma_z^{(1)}\sigma_z^{(2)} - \sigma_z^{(1)}\sigma_x^{(3)} - \sigma_z^{(2)}\sigma_x^{(3)} + \sigma_z^{(1)}\sigma_z^{(2)}\sigma_x^{(3)}) \in SU(8).$$

In this case of three identical Josephson junctions we have $E_{J_1} = E_{J_2} = E_{J_3} = E_J$, because the tunneling amplitude of the junction is a system parameter depending on the material. So this set of three Josephson junctions Hamiltonian is controlled by the parameters: $\{E_{c_1}, E_{c_2}, E_{c_3}, E_L\}$, which are called *switches*. E_{c_i} parameters are controlled by the gate voltages V_{g_i}, and E_L parameter depends on the inductor switch L. We can obtain the following eight(2^3) fundamental forms of the Hamiltonian:

$$H_1 = \frac{1}{2}E_c(\sigma_z^{(1)} + \sigma_z^{(2)} + \sigma_z^{(3)}) - \frac{1}{2}E_J(\sigma_x^{(1)} + \sigma_x^{(2)} + \sigma_x^{(3)}),\ switches : \{1, 1, 1, 0\}$$

$$H_2 = -\frac{1}{2}E_J(\sigma_x^{(1)} + \sigma_x^{(2)} + \sigma_x^{(3)}) - \frac{1}{2}E_L\sigma_y^{(1)}\sigma_y^{(2)} - \frac{1}{2}E_L\sigma_y^{(1)}\sigma_y^{(3)} - \frac{1}{2}E_L\sigma_y^{(2)}\sigma_y^{(3)},$$
$$switches : \{0, 0, 0, 1\}$$

$$H_3 = \frac{1}{2}E_c(\sigma_z^{(2)} + \sigma_z^{(3)}) - \frac{1}{2}E_J(\sigma_x^{(1)} + \sigma_x^{(2)} + \sigma_x^{(3)}),\ switches : \{0, 1, 1, 0\}$$

$$H_4 = \frac{1}{2}E_c(\sigma_z^{(1)} + \sigma_z^{(3)}) - \frac{1}{2}E_J(\sigma_x^{(1)} + \sigma_x^{(2)} + \sigma_x^{(3)}),\ switches : \{1, 0, 1, 0\}$$

$$H_5 = \frac{1}{2}E_c(\sigma_z^{(1)} + \sigma_z^{(2)}) - \frac{1}{2}E_J(\sigma_x^{(1)} + \sigma_x^{(2)} + \sigma_x^{(3)}),\ switches : \{1, 1, 0, 0\}$$

$$H_6 = \frac{1}{2}E_c\sigma_z^{(3)} - \frac{1}{2}E_J(\sigma_x^{(1)} + \sigma_x^{(2)} + \sigma_x^{(3)}),\ switches : \{0, 0, 1, 0\}$$

$$H_7 = \frac{1}{2}E_c\sigma_z^{(2)} - \frac{1}{2}E_J(\sigma_x^{(1)} + \sigma_x^{(2)} + \sigma_x^{(3)}),\ switches: \{0,1,0,0\}$$

$$H_8 = \frac{1}{2}E_c\sigma_z^{(1)} - \frac{1}{2}E_J(\sigma_x^{(1)} + \sigma_x^{(2)} + \sigma_x^{(3)}),\ switches: \{1,0,0,0\}$$

These eight Hamiltonians are linearly independent and these can generate the SU(8) algebra. Any elementary three-qubit gate is represented by a unitary 8×8 matrix $U \in SU(8)$ and t_n (is time interval corresponding to each Hamiltonians), and can be generally constructed as follows:

$$U = e^{-iH_7t_{63}}e^{-iH_6t_{62}}e^{-iH_5t_{54}}e^{-iH_4t_{53}}e^{-iH_3t_{52}}e^{-iH_2t_{51}}$$
$$e^{-iH_1t_{50}}e^{-iH_8t_{49}}e^{-iH_7t_{48}}e^{-iH_6t_{47}}\cdots e^{-iH_2t_2}e^{-iH_1t_1}$$

Next we carry out the numerical simulation for realization of *CCNOT* gate. The energy parameters are assumed as follows:

$$E_c = 2.5K,\ E_J = 0.1K,\ E_L = 0.1K$$

The efficiency of this simulation could be checked by a following test function, 'f_{test}'.

$$f_{test}(t_1, t_2, \cdots, t_{63}) = \sum_{i,j=1}^{8} |CCNOT_{ij} - (U(t_1, t_2, \cdots, t_{63}))_{ij}|^2 = ||CCNOT - U||^2$$

The problem to be solved is to find the minimum point in the 63 dimensional parameters space $\{t_1, t_2, \cdots, t_{63}\}$. We apply a minimization procedure to obtain the set of the time values, which minimize f_{test}. The result is shown in Table 1.

SUMMARY

We simulated the three-qubit device using Josephson junction, and found that it is possible that the CCNOT gate is constructed by Josephson junction. We need to develop the more efficient algorithm for the higher-order qubit. Now to realize the quantum computer, the other device based upon quantum dots and NMR are also under investigation. Cooperative phenomenona of assembled metal complexes could be adopted to realize the quantum computer in the future.

CCNOT					
H_i	*switches*	t			
H_1	{1, 1, 1, 0}	t_{01}:0.0101	t_{17}:0.0733	t_{33}:0.0954	t_{49}:0.0825
H_2	{0, 0, 0, 1}	t_{02}:0.0210	t_{18}:0.0441	t_{34}:0.0387	t_{50}:0.0743
H_3	{0, 1, 1, 0}	t_{03}:0.0733	t_{19}:0.0327	t_{35}:0.0925	t_{51}:0.0885
H_4	{1, 0, 1, 0}	t_{04}:0.0891	t_{20}:0.0713	t_{36}:0.0676	t_{52}:0.0541
H_5	{1, 1, 0, 0}	t_{05}:0.0954	t_{21}:0.0925	t_{37}:0.0493	t_{53}:0.0173
H_6	{0, 0, 1, 0}	t_{06}:0.0441	t_{22}:0.0515	t_{38}:0.0700	t_{54}:0.0006
H_7	{0, 1, 0, 0}	t_{07}:0.0825	t_{23}:0.0885	t_{39}:0.0173	t_{55}:0.0345
H_8	{1, 0, 0, 0}	t_{08}:0.0746	t_{24}:0.0298	t_{40}:0.0208	t_{56}:0.0056
H_1	{1, 1, 1, 0}	t_{09}:0.0210	t_{25}:0.0891	t_{41}:0.0441	t_{57}:0.0746
H_2	{0, 0, 0, 1}	t_{10}:0.0891	t_{26}:0.0746	t_{42}:0.0713	t_{58}:0.0517
H_3	{0, 1, 1, 0}	t_{11}:0.0441	t_{27}:0.0713	t_{43}:0.0515	t_{59}:0.0298
H_4	{1, 0, 1, 0}	t_{12}:0.0746	t_{28}:0.0517	t_{44}:0.0298	t_{60}:0.0346
H_5	{1, 1, 0, 0}	t_{13}:0.0387	t_{29}:0.0676	t_{45}:0.0700	t_{61}:0.0208
H_6	{0, 0, 1, 0}	t_{14}:0.0713	t_{30}:0.0298	t_{46}:0.0432	t_{62}:0.0247
H_7	{0, 1, 0, 0}	t_{15}:0.0743	t_{31}:0.0541	t_{47}:0.0006	t_{63}:0.0056
H_8	{1, 0, 0, 0}	t_{16}:0.0517	t_{32}:0.0346	t_{48}:0.0247	...

TABLE 1. Letter analysis of CCNOT gate

Acknowledgement

H.N. is grateful for a financial support of the Ministry of Education, Science and Culture of Japan (Research No. 12020234 and No. 13740328).

References

[1] D. Deutsch, Proc. R. Soc. London A 400, 97 (1985).

[2] D. Deutsch and R. Jozsa, Proc. R. Soc. London A 439, 553 (1992).

[3] P. W. Shor, Proc. 35nd Annual Symposium on Foundations of Computer Science, Ed. Shafi Goldwasser, IEEE Press, (1994), p124-134.

[4] Y. Nakamura, Yu. A. Pashkin, and J. S. Tsai, Nature 398, 786 (1999).

[5] K. Ch. Chatzisavvas, C. Daskaloyannis, and C. P. Panos, "Quantum machine language and quantum computation with Josephson junction", **quant-ph/0109098**.